Exercise 34

Find the derivative of the function.

$$U(y) = \left(\frac{y^4+1}{y^2+1}\right)^5$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$\begin{aligned}
U'(y) = \frac{dU}{dy} &= \frac{d}{dy}\left[\left(\frac{y^4+1}{y^2+1}\right)^5\right] \\
&= 5\left(\frac{y^4+1}{y^2+1}\right)^4 \cdot \frac{d}{dy}\left(\frac{y^4+1}{y^2+1}\right) \\
&= 5\left(\frac{y^4+1}{y^2+1}\right)^4 \cdot \frac{\left[\frac{d}{dy}(y^4+1)\right](y^2+1) - \left[\frac{d}{dy}(y^2+1)\right](y^4+1)}{(y^2+1)^2} \\
&= 5\left(\frac{y^4+1}{y^2+1}\right)^4 \cdot \frac{(4y^3)(y^2+1) - (2y)(y^4+1)}{(y^2+1)^2} \\
&= 5\left(\frac{y^4+1}{y^2+1}\right)^4 \cdot \frac{2y^5+4y^3-2y}{(y^2+1)^2} \\
&= \frac{10y(y^4+1)^4(y^4+2y^2-1)}{(y^2+1)^6}
\end{aligned}$$